

# Perturbative dissipation dynamics of a weakly driven Jaynes-Cummings system

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We generalize a microscopic master equation method to study the dissipation dynamics of Jaynes-Cummings two-level system with a weak external driving. Using perturbative analysis to extend the damping bases theory, we derive the corrected Rabi oscillation and vacuum Rabi splitting analytically. The evolution of the decoherence factor of the weakly driven system reveals that the off-diagonal density matrix elements are oscillating at a frequency dependent on the driving strength and the initial population inversion. For highly-inverted systems at the weak-driving limit, this frequency reduces to twice the value for the non-driven system, showing the dissipation dynamics unable to be discovered using more conventional approaches.

## I. INTRODUCTION

The celebrated Jaynes-Cummings (JC) model [1] predicts the splitting of the energy levels of two-level atoms under their coupling to a quantum field and provide an analytical basis for the full-quantum oscillation dynamics of two-level systems. The simple yet elegant model has paved the way to the much ramified research subject of cavity quantum electrodynamics (cQED) [2] for the atom-field interaction in optical cavities. Recently, it is incarnated in the cQED studies of superconducting circuit cavities [3, 4] where the two-level system is a superconducting qubit comprising two macroscopic persistent-current states, which has contingent applications in quantum logic gate implementation [5] for quantum computation [6].

It is well-known that one of the major challenges in realizing a functional quantum computer is the decoherence inherent to a superconducting qubit, whose sources are the multiple environmental couplings such as the electron-phonon coupling and the electron-electron coupling that results in  $1/f$  noises and phase noises innate to a solid-state system. Therefore, it is imperative to describe the qubit dynamics that accounts for the decoherence effects more accurately under the JC-model.

Earlier studies that accounts the decoherence effects employ the so-called phenomenological master equation that extends the quantum Liouville equation for the density matrix to include terms consisting of products of the density matrix and operators from a semigroup. It is the simplest implementation of Lindblad's model [7] for density matrix evolution in the sense that the coefficients of these terms responsible for decoherences are assumed a uniform constant  $\gamma$  extracted phenomenologically from experiments. Later, progresses in the direction of nonequilibrium thermodynamics [8, 9] has accumulated to the Kubo-Martin-Schwinger condition that permits the depiction of a state-dependent decay coefficient  $\gamma(\omega)$  for different terms in the master equation, where  $\omega$  is the eigenfrequency associated with the operators in the semigroup for decoherence dynamics. Consequently, the set of decoherence operators can be expanded under an exact basis of the environmental Hilbert space pertinent to a particular form of heat-bath interaction Hamiltonian, giving rise to a precise microscopic master equation derived exactly from a given environmental

coupling [10].

Here, we employ the microscopic master equation to study the quantum dynamics of a cavity QED system with the cavity mode coupled to a two-level system under the JC-model. The cavity itself is weakly driven by a resonant laser field. The decay dynamics without an external driving previously studied by Scala *et al.* shows that the populations in the quantized levels are oscillating around an averaged exponential curve derived from the phenomenological theory. When the cavity is weakly driven, we amend the exactly diagonalizing eigenvectors with perturbative terms to derive a set of nine damping bases for the Liouville superoperator, under which decay sources previously underdiscovered [11–13] are found. Using these damping bases, we find minute-scale oscillations of higher frequency on top of the original Rabi oscillations [14–16] that is enveloped by a decay due to the relaxation of the two-level system. In addition, the splitting of the dressed levels at vacuum is contracted compared to the non-driven case. More importantly, we observe the decoherence factor would follow different oscillation frequencies under different initial population inversions, which can be explained by the distinct damping bases dominating under particular inversion cases.

We first derive the microscopic master equation and the associated eigenoperators in Sec. II. Using these operators as bases, we compute the correlation functions of the cavity mode quadrature to show the effective decay rates and the noise density spectrum in Sec. III. The evolution of the level populations is shown in Sec. IV and conclusions are given in Sec. V.

## II. MICROSCOPIC MASTER EQUATION

We start the discussion with the model Hamiltonian  $H = H_{JC} + H_d$  where ( $\hbar = 1$ )

$$H_{JC} = \omega_c a^\dagger a + \frac{\omega_z}{2} \sigma_z + \Omega (a \sigma_+ + a^\dagger \sigma_-) \quad (1)$$

stands for the JC-model Hamiltonian for one two-level system of eigenfrequency  $\omega_z$  and one cavity quantum field of frequency  $\omega_c$  under the rotating wave approximation for their interaction. The weak external driving is represented by the Hamiltonian

$$H_d = \xi (a + a^\dagger). \quad (2)$$

Under weak-excitation, we consider only the lowest three eigen-levels for  $H_{JC}$  at atom-cavity resonance:  $|E_0^{(0)}\rangle = |g, 0\rangle$  and  $|E_{\pm}^{(0)}\rangle = (|g, 1\rangle \pm |e, 0\rangle) / \sqrt{2}$ . Further considering  $H_d$  as a weak perturbation to  $H_{JC}$ , we can find the perturbed eigen-basis vectors

$$|E_0\rangle = |E_0^{(0)}\rangle - \frac{\xi}{\sqrt{2}(\omega_z - \Omega)} |E_-^{(0)}\rangle - \frac{\xi}{\sqrt{2}(\omega_z + \Omega)} |E_+^{(0)}\rangle, \quad (3)$$

$$|E_{\pm}\rangle = |E_{\pm}^{(0)}\rangle + \frac{\xi}{\sqrt{2}(\omega_z \pm \Omega)} |E_0^{(0)}\rangle, \quad (4)$$

in terms of the bare basis vectors under the first-order perturbation expansion. The perturbation contributes a quadratic term correction to the eigenvalues from the driveless JC-model, which gives

$$E_0 = -\frac{\omega_z}{2} - \frac{\omega_z \xi^2}{\omega_z^2 - \Omega^2}, \quad (5)$$

$$E_{\pm} = \frac{\omega_z}{2} \pm \Omega + \frac{\xi^2}{2(\omega_z \pm \Omega)} \quad (6)$$

Pairing the vectors of Eqs. (3)-(4) with their conjugates form a set of nine damping bases [17, 18] for the Liouville operator of the master equation [10]

$$\frac{d\rho}{dt} = \mathcal{L}\rho = -i[H, \rho] + (\mathcal{L}_- + \mathcal{L}_+)\rho \quad (7)$$

where we have split the environmental contribution into two parts with each having the effect

$$\mathcal{L}_{\pm}\rho = \frac{\gamma(E_{\pm} - E_0)}{2} \times \left[ |E_0\rangle\langle E_{\pm}| \rho |E_{\pm}\rangle\langle E_0| - \frac{1}{2} \{ |E_{\pm}\rangle\langle E_{\pm}|, \rho \} \right] \quad (8)$$

on the density matrix. The damping rate  $\gamma(\Omega)$  obeys the Kubo-Martin-Schwinger (KMS) condition  $\gamma(-\Omega) = \exp\{-\Omega/k_B T\} \gamma(\Omega)$  for the quasi-equilibrium system at temperature  $T$ . Using  $\rho_{00} = |E_0\rangle\langle E_0|$  and  $\rho_{\alpha\beta} = |E_{\alpha}\rangle\langle E_{\beta}| - \delta_{\alpha\beta} |E_0\rangle\langle E_0|$  for all other  $\alpha, \beta \in \{0, +, -\}$  to denote the nine damping bases, the corresponding eigenvalues  $\lambda_{\alpha\beta}$  associated with the eigen-equation  $\mathcal{L}\rho_{\alpha\beta} = \lambda_{\alpha\beta}\rho_{\alpha\beta}$  read

$$\begin{aligned} \lambda_{00} &= 0, \quad \lambda_{++} = -\frac{\gamma_+}{2}, \quad \lambda_{--} = -\frac{\gamma_-}{2}, \\ \lambda_{0\pm} &= -\frac{\gamma_{\pm}}{4} + i \left[ \omega_z \pm \Omega + \frac{3\omega_z \mp \Omega}{2(\omega_z^2 - \Omega^2)} \xi^2 \right], \quad \lambda_{\pm 0} = \lambda_{0\pm}^*, \\ \lambda_{-+} &= -\frac{\gamma_+ + \gamma_-}{4} + i2\Omega \left( 1 - \frac{\xi^2}{\omega_z^2 - \Omega^2} \right), \quad \lambda_{+-} = \lambda_{-+}^*. \end{aligned} \quad (9)$$

where the abbreviation  $\gamma_{\pm} = \gamma(E_{\pm} - E_0)$  has been adopted. We note that since the damping basis  $\rho_{00}$  has a zero eigenvalue, it is obviously identical to the steady-state density matrix  $\rho_{ss} = \rho_{00}$  for  $e^{\mathcal{L}t}\rho_{ss} = \rho_{ss}$ . Thus, for any initial state  $\rho(0) = \sum_{\alpha,\beta} M_{\alpha\beta} \rho_{\alpha\beta}$  expanded in the damping bases, the master equation (7) admits the formal solution

$$\rho(t) = \sum_{\alpha,\beta} M_{\alpha\beta} \exp\{\lambda_{\alpha\beta} t\} \rho_{\alpha\beta}. \quad (10)$$

### III. CORRECTED RABI OSCILLATION AND VACUUM SPLITTING

The damping bases with their associated eigenvalues (9) to the master equation (7) leads to a decaying envelop to the Rabi oscillation of the two-level system. The effect of the weak driving can be examined in the temporal domain where we assume the two-level system is fully inverted initially, i.e.  $|\psi(0)\rangle = |e, 0\rangle$ . In the dressed state bases, one can expand the state to first-order in the perturbative series as

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} (|E_+\rangle - |E_-\rangle) + \frac{\xi\Omega}{\omega_z^2 - \Omega^2} |E_0\rangle, \quad (11)$$

making the corresponding density matrix  $\rho(0)$  be expandable in the damping bases as

$$\begin{aligned} \rho(0) &= \rho_{00} + \frac{1}{2} (\rho_{--} + \rho_{++} - \rho_{+-} - \rho_{-+}) \\ &\quad - \frac{\xi\Omega}{\sqrt{2}(\omega_z^2 - \Omega^2)} (\rho_{0-} - \rho_{0+} + \rho_{-0} - \rho_{+0}). \end{aligned} \quad (12)$$

The excited-state population  $P_e = \text{tr}\{|e\rangle\langle e|\rho(t)\}$  can then be computed from Eq. (10) as a function of the driving strength  $\xi$ ,

$$\begin{aligned} P_e(\xi) &= \frac{1}{4} \left( e^{-\gamma_- t/2} + e^{-\gamma_+ t/2} \right) \\ &\quad + \frac{1}{2} e^{-(\gamma_- + \gamma_+)t/4} \cos \Delta t + \frac{\xi^2 \Omega^2}{(\omega_z^2 - \Omega^2)^2} \\ &\quad \times \left[ e^{-\gamma_- t/4} \cos(\omega_- t) + e^{-\gamma_+ t/4} \cos(\omega_+ t) \right], \end{aligned} \quad (13)$$

where the first term indicates the spontaneous decay from the two dressed levels  $|E_- \rangle$  and  $|E_+ \rangle$  while the second term indicates the Rabi oscillation of population. The Rabi frequency  $\Delta = E_+ - E_-$  is corrected from its original  $2\Omega$  for a bare system by a correction term  $\xi^2 \Omega / (\omega_z^2 - \Omega^2)$  due to the driving, which makes the oscillation slower. In addition, the driving leads to a third term with  $\omega_{\pm} = E_{\pm} - E_0$ , which breaks the oscillation symmetry of the dressed levels.

Figure (1) shows the comparison of the  $P_e$  evolution over a dimensionless time scale between the non-driven systems and the driven systems, using the experimental parameters typical of a superconducting qubit where  $\omega_z = 5\text{GHz}$  and  $\Omega/\omega_z = 0.2$ ,  $\gamma_-/\omega_z = 0.002$ , and  $\gamma_+/\omega_z = 0.006$ . As one expects from Eq. (13), the non-driven case has  $P_e^{(0)}$  follow an enveloped Rabi oscillation of frequency  $\Delta$  contributed by the second term. The third term proportional to squared driving strength  $\xi^2$  has the effect of minute oscillations of higher frequencies  $\omega_+$  and  $\omega_-$  imposed on top of the Rabi oscillation, which are themselves enveloped by the decay rates  $\gamma_+$  and  $\gamma_-$  respectively. Note that the summation of the two sinusoids has the apparent effect that the minute oscillation has a carrier frequency of  $\omega_+ + \omega_-$  and an amplitude oscillation frequency of  $\omega_+ - \omega_- = \Delta$ . The first term of Eq. (13) contributes to a non-oscillating offset, which is only visible in the plot over long-term.

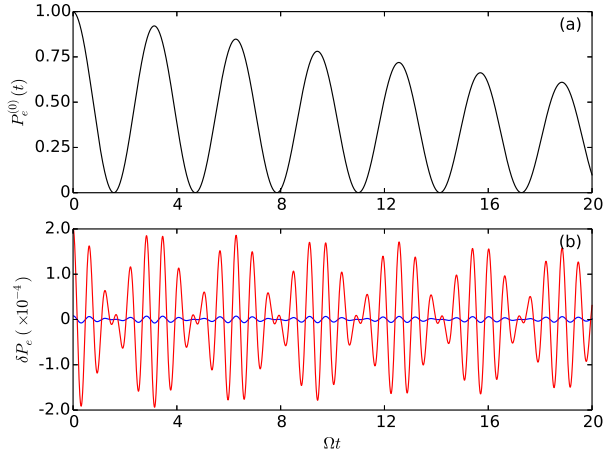


FIG. 1. (a) Plot of the excited state population  $P_e^{(0)}$  of a bare two-level system with no external driving, i.e.  $\xi = 0$ , over dimensionless time. (b) The change over  $P_e^{(0)}$  when the system is weakly driven with driving strength  $\xi/\omega_z = 0.02$  (blue curve) and  $\xi/\omega_z = 0.1$  (red curve).

To find the spectrum of the driven dressed system, we employ the so-called input-output theory, whereby the photon statistics of a probed output field is dependent on the noise input through a correlation Green function  $\langle A(\tau)B(0) \rangle$  of two operators. The time-dependent operator is meant to represent  $\langle A(\tau) \rangle = \text{tr} \{ e^{\mathcal{L}\tau} \rho(0) A \}$  and the noise density spectrum is the Fourier transform

$$S_{AB}(\omega) = \frac{1}{\pi} \Re \int_0^\infty d\tau e^{-i\omega\tau} \langle A(\tau) B(0) \rangle. \quad (14)$$

For the cavity mode considered in the JC-model, we have  $A, B \in \{a, a^\dagger\}$  so that the resulting correlation functions are essentially the perturbation-corrected fluctuation-dissipation (FD) relations of the single-mode field under the framework of microscopic master equation. At the first-order perturbative expansion given by Eqs. (3)-(4),  $|E_0\rangle$  is also an eigenvector of  $a$ , the corrections contributed by  $|E_\pm\rangle$  carrying a negligible coefficient on the order of  $\xi^2/(\omega_z^2 - \Omega^2)$ . Consequently, when we consider the system evolution begin with the equilibrium state  $\rho_{ss}$ , it becomes easy to find that

$$\begin{aligned} \langle a^\dagger(\tau) a(0) \rangle &= \text{tr} \{ e^{\mathcal{L}\tau} |E_0\rangle \langle E_0| a^\dagger a \} \\ &= -\frac{\omega_c \xi}{\omega_z^2 - \Omega^2} \text{tr} \{ e^{\lambda_{00}\tau} |E_0\rangle \langle E_0| a \} \\ &= \eta^2, \end{aligned} \quad (15)$$

where  $\eta = \xi\omega_z/(\omega_z^2 - \Omega^2)$  is a dimensionless driving-dependent quantity, which vanishes when the perturbation is taken to its weak-driving limit. Following the same approach, we can find

$$\langle a(\tau) a(0) \rangle = \langle a^\dagger(\tau) a^\dagger(0) \rangle = \eta^2, \quad (16)$$

recovering the familiar FD-relations for the cavity vacuum under the normal approach to dissipation dynamics using phenomenological theories. The difference stemming from the

microscopic approach lies in the correlation  $\langle a(\tau) a^\dagger(0) \rangle$  because  $a^\dagger$  is not an eigenvector of  $|E_0\rangle$ , rather

$$a^\dagger |E_0\rangle = \frac{1}{\sqrt{2}} (|E_+\rangle + |E_-\rangle) - \frac{\xi\omega_z}{\omega_z^2 - \Omega^2} |E_0\rangle. \quad (17)$$

Therefore,

$$\begin{aligned} \langle a(\tau) a^\dagger(0) \rangle &= \text{tr} \{ e^{\mathcal{L}\tau} |E_0\rangle \langle E_0| a a^\dagger \} \\ &= \text{tr} \left\{ \frac{a^\dagger}{\sqrt{2}} (e^{\lambda_{0+}\tau} |E_0\rangle \langle E_+| + e^{\lambda_{0-}\tau} |E_0\rangle \langle E_-|) \right\} \\ &= \frac{1}{2} [e^{\lambda_{0+}\tau} + e^{\lambda_{0-}\tau}] + \eta^2 \end{aligned} \quad (18)$$

where we have omitted the contribution from  $|E_0\rangle \langle E_0|$  in the second equality since its coefficient also vanishes in the weak-driving limit, similar to that of Eq. (15).

Collecting the four FD-relations above, we have the correlation function  $\langle x(\tau) x(0) \rangle$  for the real quadrature  $x = a + a^\dagger$  identical to Eq. (18), i.e. composing of two exponentials. Then it is straightforward to apply Eq. (14) to find each exponential contribute a Lorentzian spectral line in the frequency space, i.e.

$$\begin{aligned} S_{xx}(\omega) &= \frac{4\gamma_-}{(4\omega + 4\omega_-)^2 + \gamma_-^2} \\ &\quad + \frac{4\gamma_+}{(4\omega + 4\omega_+)^2 + \gamma_+^2} + 4\eta^2 \delta(\omega). \end{aligned} \quad (19)$$

Figure 2 shows a plot of the spectrum where we have adopted an ohmic dissipation spectrum [19]

$$\gamma(\omega) = \kappa \omega e^{-\omega/\omega_C} \quad (20)$$

with cutoff frequency  $\omega_C$  and dissipation coefficient  $\kappa$ . Then the damping channels along the two levels  $|E_+\rangle$  and  $|E_-\rangle$  become evidently unequal. With  $\omega_+ > \omega_-$  and  $\omega_C$  set to 1 GHz, we have  $\gamma_+ < \gamma_-$ , making the right Lorentzian at  $\omega_-$  in the plot exhibit a higher peak and a wider half-width than the left one at  $\omega_+$ .

The driving results in red-shifts of the two Lorentzians towards to the left-end of the plot with unequal shift amplitude. As a whole, the driving as a perturbation has the effect of reducing the vacuum Rabi split, following the formula

$$|\omega_+ - \omega_-| = 2\Omega \left[ 1 - \frac{\xi^2}{2(\omega_z^2 - \Omega^2)} \right]. \quad (21)$$

#### IV. DECOHERENCE IN DRIVEN JAYNES-CUMMINGS MODEL

The microscopic master equation approach to the driven JC model also gives a more accurate prediction of the decoherence dynamics of a qubit or two-level system in general. Consider an arbitrary initial state of the qubit and a vacuum state for the resonator, i.e.  $|\psi(0)\rangle = c_g e^{i\phi} |g, 0\rangle + c_e |e, 0\rangle$ , where we let  $c_g$  and  $c_e$  be real and the complex phase be accounted

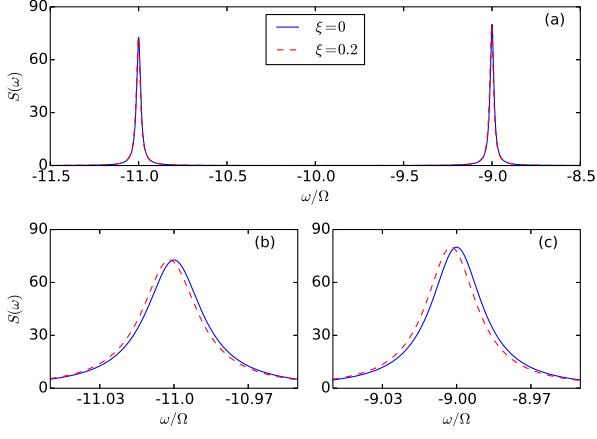


FIG. 2. (a) Plot of the spectrum  $S_{xx}$  for the weak-driving scenario  $\xi = 0.2\omega_z$  (red dashed) and the non-driven scenario  $\xi = 0$  (blue solid), showing two peaks of uneven heights and widths at  $\omega_+$  and  $\omega_-$ . Magnified views of the left and the right peaks are shown in (b) and (c), respectively.

by the phase factor  $e^{i\phi}$ . Under the truncated dressed bases, the initial state becomes

$$|\psi(0)\rangle = \left( c_g e^{i\phi} + \xi \frac{\Omega c_e}{\omega_z^2 - \Omega^2} \right) |E_0\rangle + \left( \frac{\xi c_g e^{i\phi}}{\omega_z - \Omega} - c_e \right) \frac{|E_- \rangle}{\sqrt{2}} + \left( \frac{\xi c_g e^{i\phi}}{\omega_z + \Omega} + c_e \right) \frac{|E_+ \rangle}{\sqrt{2}}. \quad (22)$$

The corresponding pure-state density matrix  $\rho = |\psi(0)\rangle \langle \psi(0)|$  can be decomposed into the damping bases  $\rho = \sum_{\alpha, \beta} M_{\alpha\beta} \rho_{\alpha\beta}$  like we did in last section and here the coefficients are

$$M_{00} = c_e c_g, \quad (23)$$

$$M_{\pm\pm} = \frac{1}{2} \left( c_e^2 \pm \frac{2\xi \cos \phi}{\omega_z \pm \Omega} c_e c_g \right), \quad (24)$$

$$M_{0\pm} = \frac{1}{\sqrt{2}} \left( \pm e^{i\phi} c_e c_g + \frac{\xi}{\omega_z \pm \Omega} c_g^2 \pm \frac{\Omega \xi}{\omega_z^2 - \Omega^2} c_e^2 \right), \quad (25)$$

$$M_{+-} = \frac{1}{2} \left( -c_e^2 - 2\xi \frac{\omega_z \cos \phi - i\Omega \sin \phi}{\omega_z^2 - \Omega^2} c_g c_e \right), \quad (26)$$

$M_{\pm 0} = M_{0\pm}^*$ , and  $M_{+-} = M_{+-}^*$ .

Consequently, through the formal solution Eq. (10), we can find the off-diagonal element  $\rho_{eg} = \text{tr}\{\rho(t) |g\rangle \langle e|\}$  of the density matrix to be

$$\begin{aligned} \rho_{eg} = & \frac{1}{\sqrt{2}} (M_{+0} e^{\lambda_{+0}t} - M_{-0} e^{\lambda_{-0}t}) \\ & + \frac{\Omega \xi}{\omega_z^2 - \Omega^2} (M_{00} e^{\lambda_{00}t} - M_{++} e^{\lambda_{++}t} - M_{--} e^{\lambda_{--}t}) \\ & + \frac{\xi}{2(\omega_z - \Omega)} (M_{+-} e^{\lambda_{+-}t} - M_{--} e^{\lambda_{--}t}) \\ & + \frac{\xi}{2(\omega_z + \Omega)} (M_{++} e^{\lambda_{++}t} - M_{+-} e^{\lambda_{+-}t}) \end{aligned} \quad (27)$$

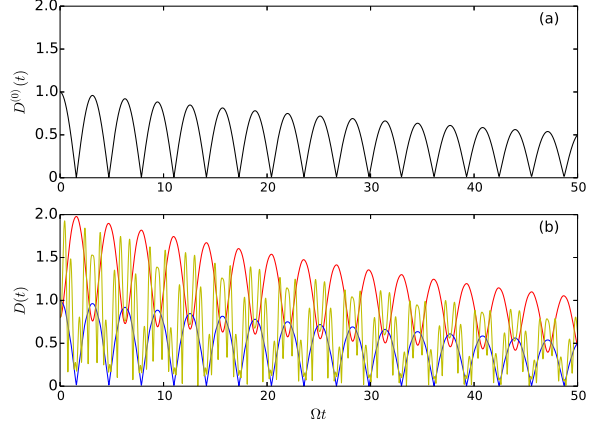


FIG. 3. (a) Plot of the decoherence factor  $D^{(0)}(t)$  as a function of time in a non-driven two-level system. (b) Plots of the decoherence factor  $D(t)$  under strong coupling  $\Omega \approx \omega_z$  and weak driving  $\xi/\omega_z = 0.1$  when the system is populated at different proportions:  $c_e/c_g = 0.1$  (red curve),  $c_e/c_g = 1$  (blue curve), and  $c_e/c_g = 100$  (yellow curve).

for calculating the decoherence factor  $D(t) = |\rho_{eg}|/c_e c_g$  [20]. Note that when the driving  $\xi$  vanishes, the terms stemmed from the perturbation in the equation above vanishes and the decoherence factor reduces to [12]

$$\begin{aligned} [D^{(0)}(t)]^2 = & \frac{1}{4} \left( e^{-\gamma_+ t/4} - e^{-\gamma_- t/4} \right)^2 \\ & + e^{-(\gamma_+ + \gamma_-)t/4} \cos^2(\Omega t), \end{aligned} \quad (28)$$

showing that the decoherence in the absence of driving is insensitive to the initial population distribution in the qubit.

Figure (3) shows the comparison of decoherence factors of the driven cases to the non-driven case, which demonstrates the inadequacy of the non-driven model where the initial population distribution plays a crucial role, particularly when the system is strongly coupled with  $\Omega \approx \omega_z$ . The parameters are typical of superconducting qubits with  $\omega_z = 5\text{GHz}$  and  $\xi/\omega_z = 0.1\omega_z$ . The decay rates  $\gamma_-/\omega_z = 0.05$  and  $\gamma_+/\omega_z = 0.055$  are chosen unequal to reflect the uneven damping channels they associated with for a given environmental reservoir, such as that of Eq. (20). The influence of population distribution is shown using three different ratios of  $c_e/c_g$  at 1 (equal distribution, blue curve), 0.1 (ground populated, red curve), and 100 (excited-state populated, yellow curve).

We observe that only when the two-level qubit is initially equally populated would the decoherence factor  $D(t)$  resemble that of a non-driven case. This shows that the driving has minimal effect on a thermally equilibrated system. On the other hand, when the system is initially either inverted or condensed to ground, the driving would set the decoherence into a more complicated oscillation cycle. From the  $c_e/c_g = 0.1$  case, one can see that the grounded population first experiences a much larger decoherence, about twice, and undergoes

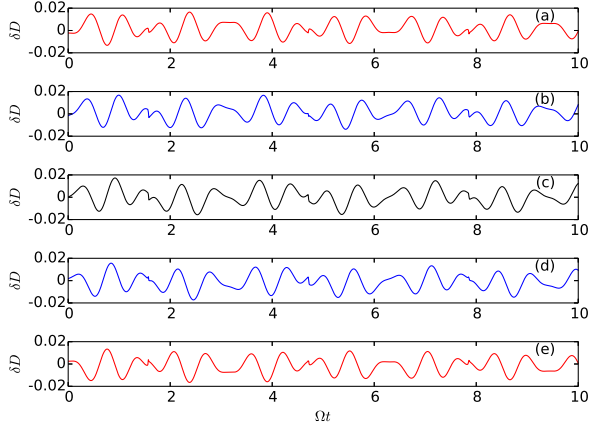


FIG. 4. Plots of the change  $\delta D = D(t) - D^{(0)}(t)$  in the decoherence factor due to different initial phases: (a)  $\phi = 0$ , (b)  $\phi = \pi/4$ , (c)  $\phi = \pi/2$ , (d)  $\phi = 3\pi/4$ , and (e)  $\phi = \pi$ .

a much quicker decay before it converges with the curve of the non-driven case. It is also noticeable that the offset, which is mainly due to the later terms in  $M_{\pm 0}$ , becomes nonzero and the phase of the decoherence oscillation is inverted from the non-driven case. Since  $c_e/c_g$  is small, the oscillation is still dominated by the terms led by the coefficients  $M_{\pm 0}$  and  $M_{00}$ , making the frequency of  $D(t)$  similar to that of  $D^{(0)}(t)$  given in Eq. (28), i.e. at  $\Omega$ .

Whereas for the inverted case with  $c_e/c_g$  large, we observe from Eqs. (24)-(26) that  $\rho_{eg}$  is dominated by  $M_{\pm\pm}$  and  $M_{+-}$  terms. Then from their associated eigenvalues  $\lambda_{\pm\pm}$  and  $\lambda_{+-}$  of the damping bases given in Eq. (9), the oscillation frequency becomes close to  $2\Omega$ . This explains the behavior of the yellow curve for  $c_e/c_g = 100$  in Fig. 3.

Moreover, one sees from Eqs. (24)-(26) that the complex phase factor  $e^{i\phi}$  of the initial state also affects the decoherence. Fig. 4 shows the change  $\delta D$  in the decoherence factor from the non-driven case  $D^{(0)}(t)$  when the phase  $\phi$  is

set to five values from 0 to  $\pi$ . We can discern clearly a period at  $\Omega t = 2\pi$  in all subplots and the inversion of decoherence between a half-period difference, that is  $\delta D(\phi = 0) = -\delta D(\phi = \pi)$ .

## V. CONCLUSIONS AND DISCUSSIONS

We have developed a weakly driven Jaynes-Cummings model under the microscopic master equation. The effects of the weak drivings are given as second-order perturbative terms in the eigenvalues associated with the damping bases of the Liouville superoperator. The exemplified effects due to the weak driving include corrected Rabi oscillations and contracted vacuum Rabi splitting. More noticeable is the sensitivity of the decoherence factor to initial population of the two-level system given the external driving, which were unaccounted for in previous non-driven JC-models. Especially when the system is initially inverted, the decoherence factor would oscillating at twice the Rabi frequency, showing the decoherence dynamics is dominated by a set of damping bases different from the usual set for its non-driven counterpart. This finding demonstrates that the microscopic master equation approach is not mere corrections to the simpler Langevin equation approach and Maxwell-Bloch equation approach for analyzing the relaxation process of a quantum system. On the contrary, it provides a better ground for analysis that leads to previous undiscovered phenomena.

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